the method of the latter, and that alone, the source of æsthetic impression. In any case the theory of Groos, which has its roots in the views of Lange and v. Hartmann, is extremely interesting and valuable, especially as contrasted with the recent psychological theory of Mr. H. R. Marshall. In the present theory, the 'self-exhibition' of which Mr. Marshall makes so much, enters as the need of impressing others with the play illusion. As to the hedonic element and its ground, however, the two theories are in sharp contrast, and that of Groos seems to me, on the whole, more adequate. In the wealth of literary reference in his book Mr. Marshall pays singularly little attention to the authors from whom Groos draws, and none to the earlier work of Professor Groos himself, but treats the play theory only in the form of Mr. Spencer's surplus energy construction. As to Groos' theory musical art would present difficulties and so would lower sensuous æsthetic effects generally.

Genetically art rests upon play, according to Herr Groos, in that the three great motives of art production, 'Self-exhibition' (*Selbstdarstellung*), 'Imitation,' and 'Decoration' (*Ausschmückung*), are found in the three great classes of animal plays, respectively, 'Courting,' 'Imitation,' and Building Art' (*Baukünste*, seen in birds' nest-building, etc.). On the strength of this, Groos finds both æsthetic appreciation and impulse in the animal, and all rests upon the original 'experimenting' impulse. Of this, however, Professor Groos does not give a satisfactory account. Experimenting is a necessary part of effective learning by 'imitation,' I think, and the use made of it in the selection of movements may be its original use.

On the whole, Professor Groos' book is both a pioneer work and one of great permanent value; it should be translated into English. It contains a good index and a full list of the literary sources.

J. MARK BALDWIN.

PRINCETON.

A Primer of the History of Mathematics. By W. W. ROUSE BALL. London, The Macmillan Co. 1895. Pp. 148, 16mo. Price, 65 cents.

A History of Elementary Mathematics, with hints on methods of teaching. By FLORIAN CAJORI. New York, The Macmillan Co. 1896. Pp. viii+304, 12mo. Price, $1.50.

The object of the 'Primer,' as well set forth in its introduction, is "to give a popular account of the history of mathematics, including therein some notice of the lives and surroundings of those to whom its development is mainly due, as well as their discoveries. Such a sketch, written in non-technical language and confined to less than 140 pages, can contain nothing beyond a bare outline of the subject, and, of course, is not intended for those to whom it is familiar." It consists of the author's larger work* reduced in size by the omission of all detailed and highly technical matter. In a few places the pruning process has been carried too far. For example, on p. 13 we are told that "after the execution of Socrates, in 399 B. C., Plato spent some years in travel * * * " but we are given no clue to the relationship of Socrates to Plato. However, the few instances of this kind which occur do not appreciably detract from the clear, well ordered and interesting style which the 'Primer' enjoys in common with its source.

The book affords to students in our high schools and colleges a means of gaining, with a small expenditure of time, a sufficiently complete history of the mathematical subjects they are studying, to give them a much greater appreciation of and interest for such subjects.

As its title indicates, Professor Cajori's book does not cover the entire field of mathematics; he restricts it to arithmetic, algebra, geometry and trigonometry, as presented in undergraduate instruction, with a short account of the history of non-Euclidean geometry. The arrangement of the material is first under the headings: 'Antiquity,' 'Middle Ages,' 'Modern Times;' under each of these are the subdivisions: 'arithmetic,' 'algebra,' 'geometry' and 'trigonometry.' For a work of its size it contains a great deal of information, and nearly every statement is supported by a reference either to original sources or to other treatises upon mathematical history. The chapters upon arithmetic are par-

*A short account of the History of Mathematics. London, the Macmillan Co. 2d edition. 1893. Pp. xxiv+520, 16mo.

ticularly rich in examples of methods of calculating which have long since disappeared from our arithmetics, and, as the author points out, some of these are, by no means, inferior to those now used. Such examples make the history of arithmetic very real to one. The sections entitled 'Causes which checked the growth of demonstrative arithmetic in England,' 'Reforms in arithmetical teaching,' and 'Arithmetic in the United States,' show forcibly the stagnation which results in regarding it not as a demonstrative science, but merely as an art of calculation.

The accounts of modern synthetic geometry and of non-Euclidean geometry (pp. 252–275) seem well chosen. It is necessary for teachers of geometry to have a broader view of their subject than is afforded by the typical text-book.

Having called attention to some of the merits of Professor Cajori's work, it is unfortunately necessary now to note some of its defects. The inconvenient method of introducing an abbreviation, the first time a work is cited, to be used for it subsequently, we trust will in future editions be remedied by a table at the end of the volume. It is confusing, if one is not certain of their identity, to have 'Ptolemy' and 'Ptolemæus' used indiscriminately. In the statement that "$\sqrt{2}$ cannot be exactly represented by any number whatever" (p. 51), the word rational has, of course, inadvertently been omitted. Foot-note 3, p. 72, is very indefinite in its present form. Referring to remarks at the top of page 74, we quite agree with the author that rigor in geometry demands the proof of the *possibility* of all constructions before they are used. For example, that the circumference of a circle admits of being divided into any number of equal parts should be shown (which involves no difficulty) before considering regular inscribed polygons in general. The example of the text leads one to suppose that rigor demands our ability to construct (subject, in fact, to the arbitrary condition of having only ruler and compass) every inscribed polygon we may wish to use.

The material of the volume in places shows lack of coordination and incomplete moulding into an organic whole. One feels at times lost in a maze of fact. We are given part of the biography of Leonardo of Pisa on page 119 and part on page 134. The origin of the word 'sine' is found on page 124 and again on page 130. On page 75 and again on page 78 we are told of the tomb of Archimedes.

In the foot-note 1, page 160, the conclusion that the base of Napier's logarithms is e^{-1} is erroneous, and it does not follow from what precedes it. If we define the logarithm of x with respect to the constant base b, by the equation $x = b^{\log x}$, then the numbers discovered by Napier are not logarithms; but if b is not restricted to be constant, the above equation defines Napier logarithms when

$$b = 10^{\frac{7}{\log x}} \div C^{\left(\frac{1}{10}\right)^7}$$

(Hagen, Synopsis der hoeheren Mathematik I., p. 107.) To define the base of Napier's logarithms as the number whose logarithm is unity is in this case misleading. The term is, however, so used by Cantor (Geschichte der Mathematik, II., p. 672), who gives its value to be

$$10^7 \div C^{\left(\frac{1}{10}\right)^7}$$

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Die Bedingungen der Fortpflanzung bei einigen Algen und Pilzen. Von DR. GEORGE KLEBS. Jena, Gustav Fischer. 1896. Pp. i.+543, 3 plates.

This work of Dr. Klebs' is an important contribution to the physiology of reproduction. As its title indicates, the experiments were conducted for the purpose of determining the conditions of reproduction in certain algæ and fungi. A preliminary account of some of this work has been published in earlier contributions. The earlier experiments have been amplified and extended to a number of additional plants, and the present work details carefully his later experiments and presents the philosophy and deductions of all his work upon this topic. It is a remarkable work, alike for the painstaking conduct of the experiments, the precautions against error, the important results obtained and the cautious generalizations upon the relations of the different kinds of reproduction to environment. Not only is the work one of great interest to the student of develop-